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THERMAL INSTABILITY OF INCOMPRESSIBLE, ROTATING FLUID SATURATED IN A POROUS MEDIUM

ARTI BANSAL[1] & NIDHI AGARWAL[2]

[1]Department of Mathematics, Inderprastha Engineering College, Ghaziabad, Uttar Pradesh, India

[2]Department of Mathematics, Satyam Group of Colleges, Pilakhuwa, Ghaziabad, Uttar Pradesh, India

ABSTRACT

The thermal instability of incompressible, rotating fluid saturated in a porous medium is critically examined both analytically and numerically within the framework of linear analysis.. The analytical discussion provides the sufficient conditions of stability and instability and the characterization of modes. By actually calculating the root of eigenvalue equation (of degree 3) neutral stability curves are drawn. The numerical results show the effect of various physical parameters on the critical wave number a_c. The important conclusion is that medium porosity parameter R_D^{-1} and Prandtl number P_r show a dual character . The role of other physical parameters such as rotation parameter T , Richardson number J and non-dimensional parameter R_2 has also been discussed in the paper.

KEYWORDS: Thermal Instability, Porous Medium, Rotating Fluid

INTRODUCTION

In recent years, there has been considerable interest among geophysical fluid dynamicists to study the flows and the flow instabilities in porous medium because of numerous scientific and industrial applications. The instability of fluid flows in a porous medium under varying assumptions has been well summarized by Scheidegger (1960) and Yih (1961). While investigating the flows or flow instabilities through porous media, the liquid flow has been assumed to be governed by Darcy's Law (1856) by most of the research workers, which neglects the inertial forces on the flow.

Brinkman (1947) proposed a plausible modification to Darcy's law. He argued that the equation in a porous medium must reduce to the ordinary Navier-Stoke's equation, in the limit of zero permeability. This led to the celebrated Brinkman's model. He proposed the introduction of the term $-\left(\frac{\mu}{k_1}\right)\boldsymbol{V}$ in addition to $\frac{\mu}{\varphi}\nabla^2\boldsymbol{V}$ in the equations of fluid motion.

Instability of compressible or incompressible flow has been studied extensively by a number of research workers in past few decades. In almost all such investigations, the Boussinesq approximation is used to simplify the equations of motion. In an important paper, Goel *et.al.* (1999) have examined both numerically and analytically the hydromagnetic stability of an unbounded couple stress binary fluid mixture having vertical temperature and concentration gradients with rotation.

Bansal and Agrawal (1999) examined the shear flow instability of an incompressible visco-elastic fluid in a porous medium in the presence of a weak magnetic field.

The important results obtained include the extension of sufficient condition of stabilities of Goel *et.al.* (1997) to the hydromagnetic case, characterization of modes and bounds on the complex wave velocity of unstable modes.

In this paper, An attempt has been made to study the thermal instability of incompressible, rotating fluid saturated in a porous medium. Following Brinkman, the porosity term is included in the equation of motion.

Important analytical results obtained in this paper include the sufficient conditions for stability and instability, we have actually solved the eigen-value equation of degree three obtaining all the three roots which, in general, are complex. On the basis of this data, curves have been plotted to provide neutral stability curves demonstrating clearly the role of various physical parameters.

FORMULATION OF THE PROBLEM

An incompressible, heat conducting, viscous fluid, saturating in an isotropic porous medium, is confined in the infinite space $-\infty < x, y, z < \infty$ having z-axis vertically upward so that the gravity acts in the negative z-direction and is subjected to uniform rotation Ω about z-axis chosen in the direction of gravity. Uniform temperature is maintained along z-axis.

Following Brinkman, the governing equations in a rectangular frame of reference (x,y,z) rotating with fluid, with porosity corrections are:

$$\frac{\rho}{\phi}\left[\frac{\partial \boldsymbol{v}}{\partial t}+\frac{1}{\phi}(\boldsymbol{v}.\nabla)\boldsymbol{v}\right]=-\nabla p+\rho\boldsymbol{g}+\mu\left(\frac{1}{\phi}\nabla^2-\frac{1}{k_1}\right)\boldsymbol{v}+\frac{2\rho}{\phi}(\boldsymbol{v}\times\boldsymbol{\Omega}) \,, \tag{1}$$

$$\nabla.\boldsymbol{v}=0, \tag{2}$$

$$\frac{\partial T}{\partial t}+\frac{1}{\phi}(\boldsymbol{v}.\nabla)T=\kappa\nabla^2 T \tag{3}$$

and

$$\rho=\rho_0\left[1-\alpha(\mathrm{T}-\mathrm{T}_0)\right], \tag{4}$$

Where $\boldsymbol{v}$, ρ , μ , T, κ; α, ϕ and k_1 are respectively fuid velocity, density, dynamic viscosity coefficient, temperature, thermal diffusivity, thermal expansion coefficient, medium porosity, and medium permeability. $g = (0,0,-g)$ is gravitational acceleration and the suffix zero refers to the values at the reference level $z=0$.

The basic state for the problem under consideration is characterized by

$$\left.\begin{aligned}&\boldsymbol{v}=(0,0,0),\\&T=T_0-\beta z,\\&\boldsymbol{\Omega}=(0,0,\Omega),\end{aligned}\right] \tag{5}$$

Where β may be either positive or negative.

The condition for equilibrium requires that

$$\frac{\partial p}{\partial z}+\rho g=0\ , \tag{6}$$

$$\rho=\rho_0[1+\alpha\beta z] \tag{7}$$

and $$p=p_0\text{-}\rho_0 g\left[z+\frac{1}{2}\alpha\beta z^2\right] \tag{8}$$

PERTURBATIONS AND NORMAL MODE ANALYSIS

The basic state characterized by eq.(8) is slightly perturbed, equations in perturbations are linearized within the framework of linear analysis, arbitrary perturbation *f'(x,y,z,t)* is decomposed into wave-like components as

$$f'(x,y,z,t)=f\exp[\sigma t+i(ax+by+cz)] \tag{9}$$

where *a*, *b* and *c* are the real wave numbers and σ, a time constant, is complex, in general and after eliminating various physical quantities, the following eigen-value equation is obtained :

$$(\sigma+\kappa l^2)\left[l^2\left\{\frac{\sigma\rho_0}{\phi}+\mu\left(\frac{l^2}{\phi}+\frac{1}{k_1}\right)\right\}^2+\frac{4\Omega^2\rho_0^2c^2}{\phi^2}\right] \tag{10}$$

Where $l^2=a^2+b^2+c^2$ and $m^2=a^2+b^2$.

Introducing the transformations

$$\sigma=\frac{U_0\sigma^*}{d}\ \text{And}\ (l,m,c)=\frac{1}{d}(l^*,m^*,c^*), \tag{11}$$

We get the following eigen-value equation (after dropping the asterisks), in non-dimensional form as

$$\sigma^3+A\sigma^2+B\sigma+C=0, \tag{12}$$

Where $A=(1+2P_r)R_2l^2+2R_D^{-1}$,

$$B=(2+P_r)P_rR_2^2l^4+2R_2(1+P_r)l^2R_D^{-1}+R_D^{-2}+\frac{Tc^2}{l^2}-\frac{Jm^2}{l^2}$$

$$C=P_r^2R_2^3l^6+2R_2^2P_rR_D^{-1}l^4+R_2R_D^{-2}l^2+R_2Tc^2-Jm^2P_rR_2-\frac{Jm^2R_D^{-1}}{l^2},$$

$$\nu=\frac{\mu}{\rho_0},\ P_r=\frac{\nu}{\kappa},\ R_2=\frac{\kappa}{dU_0},\ R_D^{-1}=\frac{\nu d\phi}{k_1U_0},$$

$$J=\frac{g\alpha\beta d^2}{U_0^2}\ \text{And}\ T=\frac{4\Omega^2d^2}{U_0^2}$$

It is important to note that the values of J can be both positive and negative depending upon whether the temperature decreases or increases in the vertically upward direction.

ANALYTICAL DISCUSSIONS

In this section, we shall prove some important results with the help of equation (12).

Case-I: C < 0

C is negative under the condition

$$J\, m^2\left(P_r R_2\, l^2 + R_D^{-1}\right) \;>\; l^2\left(P_r^2 R_2^3 l^6 + 2\, P_r R_2^2 R_D^{-1} l^4 + R_2\, R_D^{-2} l^2 + R_2 T\, c^2 \right) \tag{13}$$

For this condition to be true, it is necessary that the temperature must decrease in the vertically upward direction. It is well known that this decrease of temperature ($J > 0$) in the vertically upward direction promotes instability.

Theorem1:- System is unstable under the condition $C < 0$.

Proof: - If C < 0, the product of roots of the equation (12) is positive so that at least one positive root exists, implying, thereby, the instability of the system.

Theorem 2:- The unstable modes which exists under the condition $C < 0$ are non-oscillatory.

Proof: - The imaginary part of equation (12) after its division by σ yields

$$\sigma_i \left\{ 2\sigma_r + A - \frac{C}{|\sigma|^2} \right\} \;=\; 0$$

Since A is positive and in view of theorem -1, modes will be unstable when $C < 0$. Therefore the above equation can be written as

$$\sigma_i \left\{ 2\sigma_r + A + \frac{|C|}{|\sigma|^2} \right\} \;=\; 0 \tag{14}$$

It follows that σ_i must necessarily be zero so that the unstable modes are non-oscillatory.

Theorem -1 and 2 when combined together yield that if $C < 0$, then the modes are unstable and non-oscillatory.

Case-II : C > 0

Theorem 3:- The non-oscillatory modes are stable, if $B > 0$.

Proof: - Equation (12) for non-oscillatory modes ($\sigma_i = 0$) reduces to

$$\sigma_r^3 + A\sigma_r^2 + B\,\sigma_r + C \;=\; 0$$

Since A, B and C are all positive, σ_r should be negative, implying, thereby, the stability of non-oscillatory modes, if exist in the system.

NUMERICAL RESULTS AND DISCUSSIONS

Equation (12) is a cubic equation in σ with real coefficients which depend upon R_D^{-1}, P_r, J, R_2, T and the wave numbers *a*, *b* and *c* (through *l* and *m*). We have calculated all the three roots of this equation. Our aim has been to examine quantitatively the effect of various parameters such as the Prandtl number P_r, Darcy-Reynolds number R_D^{-1}, Richardson number *J*, rotation parameter *T* and the parameter R_2. This has been achieved by actually calculating the value of critical wave number for different values of above parameters.

Figure 1: Shows that dual character of permeability parameter R_D^{-1}. For $J = 5$, as R_D^{-1} increases upto 2.75, the range of stable wave numbers decreases and the range of stable wave numbers increases as R_D^{-1} further increases beyond 2.75. This figure also shows a destabilizing character of *J* as for a fixed value of R_D^{-1}, the critical wave number a_c increases as *J* increases. This fact is also evident from Figure *2*, *3* and *5*.

Figure 2: Shows the effect of Prandtl number P_r on critical wave number a_c. This figure shows the dual character of Prandtl number P_r. For $J = 5$, as P_r increases from 0 to 3.25 approximately, the range of stable wave numbers decreases, however, the range of stable wave numbers increases as P_r further increases beyond 3.50.

Figure 3: Shows the critical wave number a_c for different values of rotation parameter *T*. As *T* increases, a_c decreases so that the range of stable wave numbers increases. It is concluded that *T* has a stabilizing character and a large value of *T* is required to stabilize the system for all wave numbers.

Figure 4: Shows the destabilizing character of *J*. As *J* increases, a_c increases which decreases the range of stable wave numbers.

Stabilizing character of non-dimensional parameter R_2 is exhibited in *Fig.5* . As R_2 increases, the range of stable wave numbers increases.

CONCLUSIONS

The analytical discussion provides the sufficient conditions of stability and instability and the characterization of modes. The numerical results show the effect of various physical parameters on the critical wave number a_c. On the basis of numerical discussion & neutral stability curves obtained in paper, it is concluded that medium porosity parameter R_D^{-1} and Prandtl number P_r show a dual character. The rotation parameter *T* and non-dimensional parameter R_2, which depends upon thermal diffusivity κ; have stabilizing character and Richardson number *J* has destabilizing character.

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REFERENCES

1. Scheidegger A. E., Growth of instabilities on displacement fronts in porous media, Phys Fluids, (1960) 3 94.
2. Yih C. S., Flow of a non-homogeneous fluid in a porous medium, J. Fluid Mech. (1961), 10 133.

3. H. Darcy, Les fontaines publiques de la ville de Dijon Paris : Victor Dalmont (1856).

4. Brinkman H. C., A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles, Appl. Sci. Res., (1947) A1 27.

5. Goel A. K., Agrawal S. C. and Agarawal G.S., Hydromagnetic stability of an unbounded couple stress binary fluid mixture having vertical, temperature and concentration gradients with rotation, Indian J. pure appl. Math.(1999) 30(10) 991.

6. Bansal N., Jaimala and Agrawal S. C., Shear flow instability of an incompressible visco-elastic fluid in a porous medium in the presence of ,a weak magnetic field. Indian J. pure appl. Math.(1999), 30(12) 1283.

7. Goel A. K., Agrawal S. C. and Jaimala, shear flow instability of an incompressible visco- elastic second order fluid in a porous medium,Indian J. pure appl. Math. (1997) 28(4) 563.\

APPENDICES

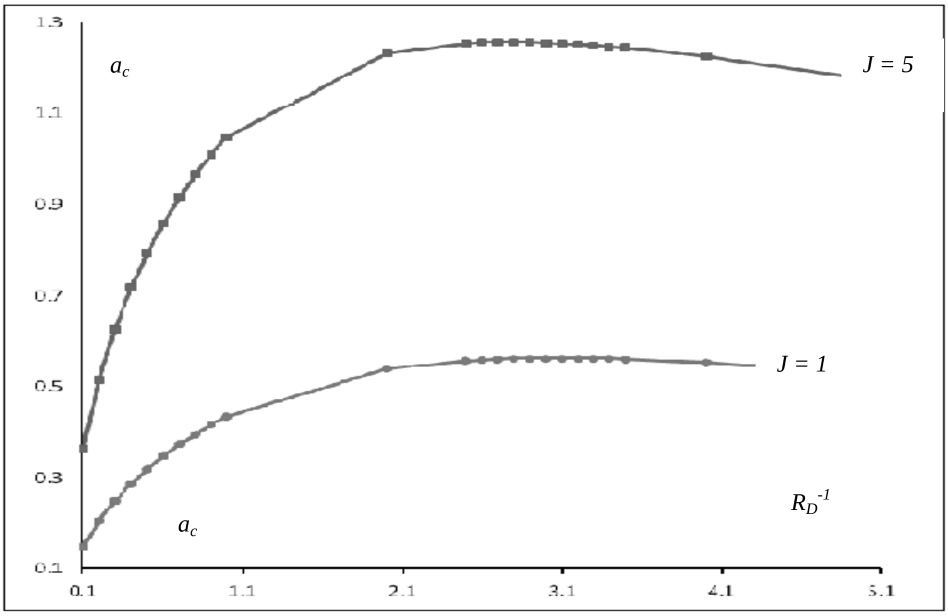

Figure 1: Critical Wave Number A_c Vs R_D^{-1} (R2 = P_r = 0.5, T = 10)

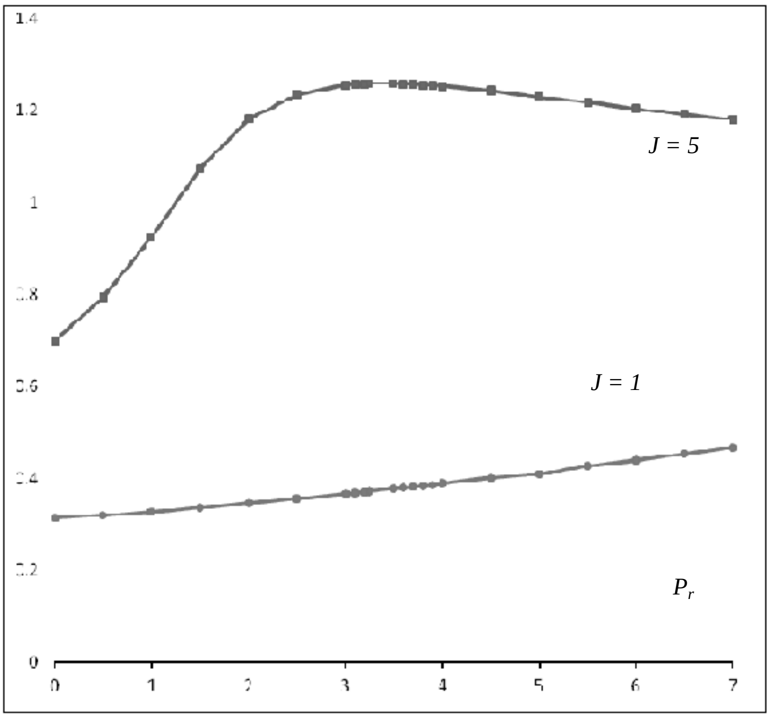

Figure 2: Critical Wave Number A_c vs P_r ($R_2 = R_D^{-1} = 0.5$, T = 10)

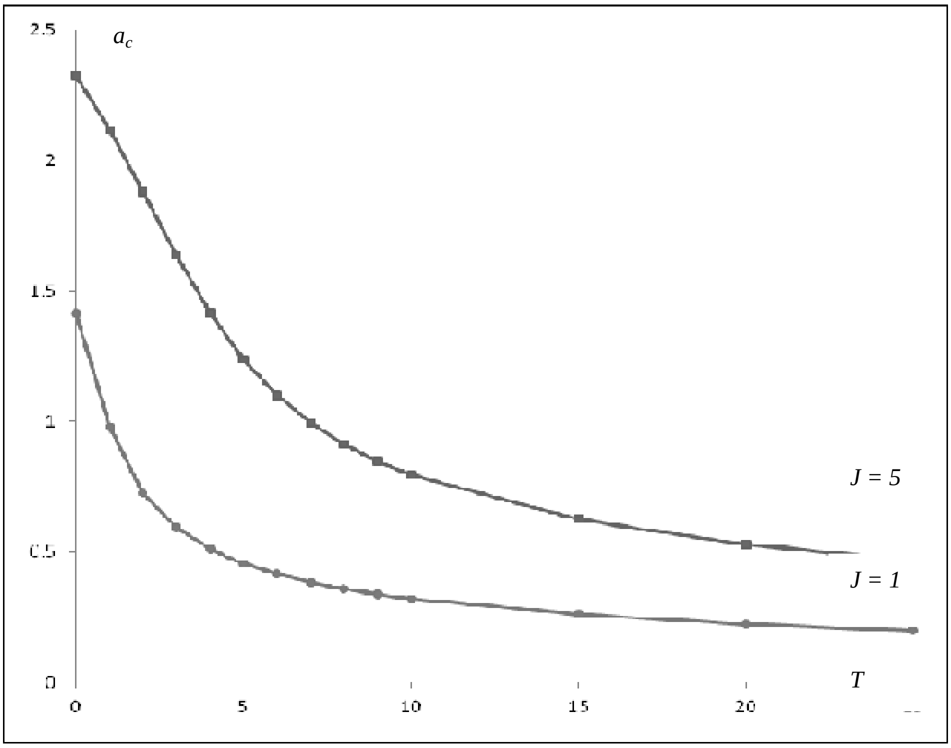

Figure 3: Critical Wave Number A_c vs T (R2 = $R_D^{-1} = P_{r} = 0.5$)

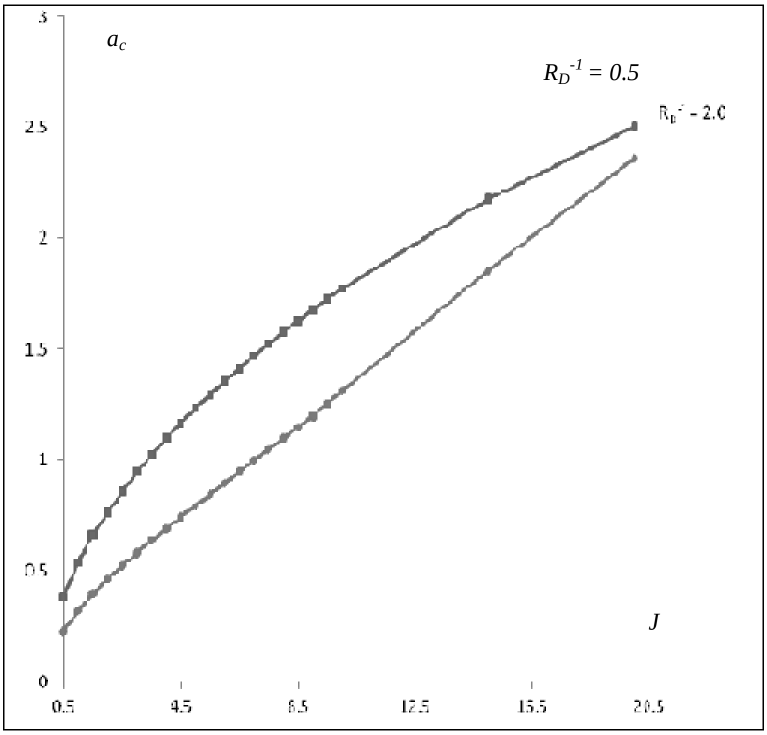

Figure 4: Critical Wave Number A_c vs J (R_2 = P_r = 0.5, T = 10)

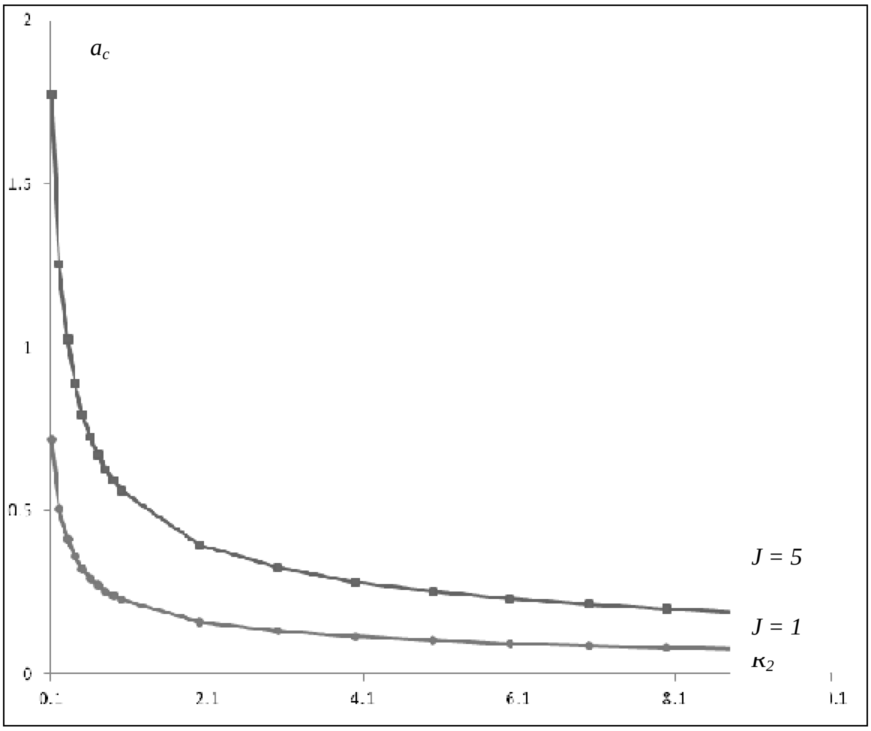

Figure 5: Critical Wave Number A_c vs R_2 (RD^{-1}= P_r = 0.5, T = 10)